MTH 234 - Quiz 2

Due 29 May at the beginning of class

Name:

You may work together on solving these problems, but what you hand in must be your work written in your own words.

1. (5 points) If \vec{r} is a twice-differentiable vector-valued function, show that

$$\frac{d}{dt}[\vec{r}(t) \times \vec{r}'(t)] = \vec{r}(t) \times \vec{r}''(t)$$

2. (5 points) If $\vec{r}(t) \neq 0$, show that

$$\frac{d}{dt}|\vec{r}(t)| = \frac{1}{|\vec{r}(t)|}\vec{r}(t) \cdot \vec{r}'(t)$$

Hint: One way is to write $|\vec{r}(t)|^2 = \vec{r}(t) \cdot \vec{r}(t)$ and differentiate both sides with the chain rule.

- 3. (5 points) Consider the function $\vec{r}(t) = \langle t, \ln t, t \rangle, t > 0.$
 - (a) Compute the curvature κ of \vec{r} .
 - (b) What is $\lim_{t\to\infty} \kappa(t)$? Describe this geometrically.

4. (5 points) A particle moves along the helix $x(t) = 3\cos t$, $y(t) = 3\sin t$, z(t) = t, starting at time 0 (at the point (3,0,0)). How long does it take for the particle to travel 1 unit?